Interaction between rock deformation and fluid flow

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ask any questions at anytime

2002 M7.8 Denali earthquake, 3100 km away
Some observations

• Water levels correlated with Earth tides
• Water levels go up and down as a freight trains pass
• Subsidence of the land after fluid extraction (oil, gas, water)
• Water levels rise near a pumping well (Noordergum effect)
• Filling reservoirs (e.g., Lake Mead 1935) triggers 100s of earthquakes

List based on Wang (2000)
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Deformation-fluid flow interactions: Manga

Domenico and Schwartz (1998)
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Deformation-fluid flow interactions: Manga

Shirzaei et al., Science (2016)
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List based on Wang (2000)
Reservoir-induced seismicity - surface loading
Internal loading

- waste fluid disposal
- carbon sequestration
- geothermal energy

Rocky Mountain Arsenal, CO

Deformation-fluid flow interactions: Manga
Two basic phenomena
Solid-to-fluid coupling: stress produces change in fluid pressure
Fluid-to-solid coupling: change in fluid pressure changes volume of solids

• How are deformation and fluid pressure coupled?
• How to couple deformation and fluid flow?
• What can we do with this understanding?

Notes available (too many equations)
1. Biot (1941)
Constitutive relations for isotropic stress

Saturated and isothermal rock
Stress and pore pressure an independent variables and we would like to know strain and changes in fluid content $f$, the volume of fluid *transported* in or out of storage (fluid mass/unit volume divided by fluid density)

$$\epsilon = \epsilon(\sigma, p) \quad f = f(\sigma, p)$$

If stress is isotropic then

$$d\epsilon = \left(\frac{\partial \epsilon}{\partial \sigma}\right)_p d\sigma + \left(\frac{\partial \epsilon}{\partial P}\right)_\sigma dp$$

$$df = \left(\frac{\partial f}{\partial \sigma}\right)_p d\sigma + \left(\frac{\partial f}{\partial P}\right)_\sigma dp$$
1. Biot (1941)
Constitutive relations for isotropic stress

\[ \frac{1}{K} = \left( \frac{\partial \epsilon}{\partial \sigma} \right)_p, \quad \frac{1}{H} = \left( \frac{\partial \epsilon}{\partial p} \right)_\sigma, \quad \frac{1}{H_1} = \left( \frac{\partial f}{\partial \sigma} \right)_p, \quad \frac{1}{R} = \left( \frac{\partial f}{\partial p} \right)_\sigma \]

compressibility

\[ d\epsilon = \frac{1}{K} d\sigma + \frac{1}{H} dp \]
\[ df = \frac{1}{H_1} d\sigma + \frac{1}{R} dp \]

Biot (1941) argued why \( H = H_1 \)

\( 1/R \) specific storage coefficient at constant stress
\( 1/H \) poroelastic expansion coefficient
1. Biot (1941)
Related poroelastic constants

Skempton’s coefficient

\[ B = - \left( \frac{\partial p}{\partial \sigma} \right)_f = \frac{R}{H} \]

Change is pressure/change in stress as constant fluid mass

Biot-Willis coefficient

\[ \alpha = \frac{df}{d\epsilon} |_{dp=0}, \quad \alpha = \frac{K}{H} \]

Change is fluid content/strain at constant pressure
1. Biot (1941)
Related poroelastic constants

Storage properties (change in fluid content with changes in pressure) depend on conditions

\[ S_\sigma = \left( \frac{\partial f}{\partial p} \right)_\sigma = \frac{1}{R} \]

\[ S_\varepsilon = \left( \frac{\partial f}{\partial p} \right)_\varepsilon = S_\sigma - \frac{K}{H^2} \]
1. Biot (1941)

Other forms of constitutive laws

\[ d\sigma = \left( \frac{K}{1 - \alpha B} \right) d\varepsilon - \left( \frac{K}{1 - \alpha B} B \right) df \]

\[ dp = -\left( \frac{K}{1 - \alpha B} B \right) d\varepsilon + \left( \frac{K}{1 - \alpha B} \frac{B}{\alpha} \right) df \]

\[ K_u = \frac{d\sigma}{d\varepsilon} \text{ for } f = 0, \]

\[ K_u = \frac{K}{1 - \alpha B} \]

Thus

\[ d\sigma = K_u d\varepsilon - K_u B df \]

Rearrange

\[ d\varepsilon = \frac{d\sigma}{K_u} + B df \]

Strain has two parts: first is elastic for undrained conditions, second is from fluid transfer
1. Biot (1941)

Using wells as strain meters

\[ dp = -K_u B d\epsilon + \frac{K_u B}{\alpha} df \]

\[ dh = \frac{1}{\rho_w g} p|_{f=0} = -\frac{K_u B}{\rho_w g} d\epsilon \]
1. Biot (1941)
Concept of effective stress

\[ d\varepsilon = \frac{1}{K} (d\sigma + \frac{K}{H} dp) = \frac{1}{K} (d\sigma + \alpha dp) = \frac{1}{K} d\sigma' \]

\[ d\sigma' = d\sigma + \alpha dp. \]
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List based on Wang (2000)
3 causes for failure

1. Deformation
2. Fluid flow interaction
3. Manga

Deformation and fluid flow interaction: Manga
Hubbert and Rubey (1959)
Internal loading from Hsieh and Bredehoeft (1981)

Rocky Mountain Arsenal, CO

Waste fluid injected

Seismicity well plume fluid injection

- Waste fluid disposal
- Carbon sequestration
- Geothermal energy


Deformation-fluid flow interactions: Manga
Shirzaei et al., *Science* (2016)
2. Constitutive relations for anisotropic stress

\[
d\varepsilon_{xx} = \frac{1}{E} d\sigma_{xx} - \frac{\nu}{E} d\sigma_{yy} - \frac{\nu}{E} d\sigma_{zz} + \frac{dp}{3H} \\
d\varepsilon_{yy} = -\frac{\nu}{E} d\sigma_{xx} + \frac{1}{E} d\sigma_{yy} - \frac{\nu}{E} d\sigma_{zz} + \frac{dp}{3H} \\
d\varepsilon_{zz} = -\frac{\nu}{E} d\sigma_{xx} - \frac{\nu}{E} d\sigma_{yy} + \frac{1}{E} d\sigma_{zz} + \frac{dp}{3H} \\
d\varepsilon_{xy} = \frac{1}{2G} d\sigma_{xy} \\
d\varepsilon_{yz} = \frac{1}{2G} d\sigma_{yz} \\
d\varepsilon_{xz} = \frac{1}{2G} d\sigma_{xz} \\
df = \frac{1}{H} d\sigma + \frac{1}{R} dp
\]

In standard index notation

\[
\varepsilon_{ij} = \frac{1}{2G} \left( \sigma_{ij} - \frac{\nu}{1+\nu} \sigma_{kk}\delta_{ij} \right) + \frac{p}{3H} \delta_{ij}
\]

\[
G = \frac{E}{2(1+\nu)}
\]

equivalently

\[
\sigma_{ij} = 2G \left( \varepsilon_{ij} + \frac{\nu}{1-2\nu} \varepsilon_{kk}\delta_{ij} \right) - \alpha p \delta_{ij}
\]
3. Poroelastic constants

Very many because there are many possible constraints on the REV

3.1 Compressibility

unjacketed

\[
\frac{1}{K_s'} = -\frac{1}{V} \left( \frac{\delta V}{\delta p} \right)_{p_d=0} \quad \text{and} \quad \frac{1}{K_\phi} = -\frac{1}{V_p} \left( \frac{\delta V_p}{\delta p} \right)_{p_d=0}
\]

drained

\[
\frac{1}{K} = -\frac{1}{V} \left( \frac{\delta V}{\delta p_c} \right)_{p=0} \quad \text{and} \quad \frac{1}{K_p} = -\frac{1}{V_p} \left( \frac{\delta V_p}{\delta p_c} \right)_{p=0}
\]

It is possible (see notes) to write all these in terms of \( K, K_f, \alpha, B \) and \( \phi \).
3. Poroelastic constants

3.2 Storage capacity

Undrained specific storage

\[ S_\sigma = \left( \frac{\partial f}{\partial p} \right)_\sigma = \frac{1}{R} = \frac{\alpha}{KB} \]

Constrained specific storage

\[ S_\varepsilon = \left( \frac{\partial f}{\partial p} \right)_\varepsilon = S_\sigma - \frac{K}{H^2} = S_\sigma - \frac{\alpha^2}{K} \]

Uniaxial specific storage

\[ S_s = \rho_f g \left( \frac{\partial f}{\partial p} \right)_{\sigma_{zz}=0, \varepsilon_{xx}=\varepsilon_{yy}=0} \]
3. Poroelastic constants

3.4 Coefficients of undrained pore pressure buildup

If no horizontal strains, define loading efficiency as

$$\gamma = - \left( \frac{\partial p}{\partial \sigma_{zz}} \right)_{\varepsilon_{xx}=\varepsilon_{yy}=0, f=0}$$

$$\gamma = \frac{B}{3} \frac{1 + \nu_u}{1 - \nu_u}$$

Tidal efficiency is the change in water level near the ocean

$$T.E. = \gamma = \frac{\alpha}{K_v S}$$

Barometric efficient is response to atmospheric pressure, which loads both the surface and the water in the well

$$B.E. = 1 - \gamma$$

Measurements of T.E. and B.E. can be used to determine S and $\phi$. 

Deformation-fluid flow interactions: Manga
Barometric efficiency $B.E. = 0.73$

Incompressible fluid and solid $B = 1$
Infinitely compressible fluid $B = 0$
4. Governing equations for fluid flow

Conservation of mass

\[
\frac{\partial f}{\partial t} = -\nabla \cdot \mathbf{q} + Q
\]

Combine with Darcy’s law

\[
\frac{\partial f}{\partial t} = \frac{k}{\mu} \nabla^2 p + Q
\]

\ldots And then use all the various and appropriate previous expression to relate \( f \) to stress, strain and \( p \)

Uniaxial strain and constant vertical stress

\[
f = Sp
\]

\[
S \frac{\partial p}{\partial t} = \frac{k}{\mu} \nabla^2 p + Q
\]

This is the “standard” groundwater flow equation in hydrogeology
In general, flow creates strain

\[ f = \frac{\alpha}{K} \frac{\sigma_{kk}}{3} + \frac{\alpha}{KB} p \]

\[ \frac{\alpha}{KB} \left[ \frac{B}{3} \frac{\partial \sigma_{kk}}{\partial t} + \frac{\partial p}{\partial t} \right] = \frac{k}{\mu} \nabla^2 p + Q \]

Changes in mean stress looks like a source of fluid
5. Permeability changes?
Approach: use response to solid Earth tides

Model of a well-aquifer system
(Hsieh et al., 1987)

$h_f$: ‘pressure head’ in the aquifer away from the well

$s$: ‘drawdown’ near the well

Assume constant vertical stress, only vertical strain

Horizontal flow, homogeneous, isotropic aquifer

Deformation-fluid flow interactions: Manga
Hydraulic head variations assumed to be periodic

\[ h = h_0 \exp(i\omega t) \]

With water level response

\[ x = x_0 \exp(i\omega t) \]

Groundwater flow equation

\[
\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} - \frac{S}{T} \frac{\partial s}{\partial t} = 0.
\]

with boundary conditions

\[
2\pi r_w T \left( \frac{\partial s}{\partial r} \right)_{r=r_w} = -Q_0 \exp(i\omega t) \text{ at } r = r_w
\]

\[
r \to \infty, \ s \to 0.
\]

Since equation is linear and forcing is harmonic

\[
s(r, t) = G(r) \exp(i\omega t)
\]

\[
\frac{d^2 G}{dr^2} + \frac{1}{r} \frac{dG}{dr} - \frac{i\omega S}{T} G = 0
\]

Deformation-fluid flow interactions: Manga
Solution

\[ s_w = -\frac{\omega r_c^2 x_0}{2T} \left[ (\Psi \text{Ker}(\alpha_w) + \Phi \text{Kei}(\alpha_w)) - i(\Phi \text{Ker}(\alpha_w) - \Psi \text{Kei}(\alpha_w)) \right] \exp(i\omega t) \]

\[ \Phi = -\frac{\text{Ker}_1(\alpha_w) + \text{Kei}_1(\alpha_w)}{\sqrt{2\alpha_w}[\text{Ker}_1^2(\alpha_w) + \text{Kei}_1^2(\alpha_w)]} \]

\[ \Psi = -\frac{\text{Ker}_1(\alpha_w) - \text{Kei}_1(\alpha_w)}{\sqrt{2\alpha_w}[\text{Ker}_1^2(\alpha_w) + \text{Kei}_1^2(\alpha_w)]} \]

\[ \alpha_w = \left( \frac{\omega S}{T} \right)^{1/2} r_w \]
With amplitude ratio and phase

\[ A = \frac{x_0}{h_0} = (E^2 + F^2)^{-1/2} \]

\[ \eta = -\tan^{-1}(F/E) \]

\[ E = 1 - \frac{\omega r_c^2}{2T} \left[ \Psi \text{Ker}(\alpha_w) + \Phi \text{Kei}(\alpha_w) \right] \]

\[ F = \frac{\omega r_c^2}{2T} \left[ \Phi \text{Ker}(\alpha_w) i \Psi \text{Kei}(\alpha_w) \right] \]
Hsieh et al., *JGR* (1987)
Hsieh et al., *JGR* (1987)
Equivalent problem for vertical flow to a free surface (distance $\delta$)

$$A = \frac{1}{S} \left[ 1 - 2 \exp(-z/\delta) \cos(z/\delta) + \exp(-2z/\delta) \right]^{1/2}$$

$$\eta = \tan^{-1} \left[ \frac{\exp(-z/\delta) \sin(z/\delta)}{1 - \exp(-z/\delta) \cos(z/\delta)} \right]$$

$$\delta = \sqrt{2T/\omega}.$$

**Table 2. Tide Constituents Appropriate for Analysis of Approximately Two Months of Water Level Data**

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Period (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>26.86840</td>
</tr>
<tr>
<td>O1</td>
<td>25.81930</td>
</tr>
<tr>
<td>NO1</td>
<td>24.83320</td>
</tr>
<tr>
<td>P1</td>
<td>24.06590</td>
</tr>
<tr>
<td>S1</td>
<td>24.00000</td>
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<tr>
<td>K1</td>
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<tr>
<td>J1</td>
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<tr>
<td>OO1</td>
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<tr>
<td>MU2</td>
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<td>N2</td>
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<tr>
<td>M2</td>
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<tr>
<td>L2</td>
<td>12.19160</td>
</tr>
<tr>
<td>S2</td>
<td>12.00000</td>
</tr>
<tr>
<td>K2</td>
<td>11.96720</td>
</tr>
</tbody>
</table>
Looking for tectonic and earthquake signals

• Need to correct for atmospheric pressure changes
• Account for tides
Raw data corrected for tides and barometric pressure
Coseismic changes of phase shift of the water level in response to the $M_2$ tide in two wells in southern California (Elkhoury et al. *Nature* 2006). Vertical dashed lines show the time of occurrences of earthquakes.

Deformation-fluid flow interactions: Manga
Two basic phenomena
Solid-to-fluid coupling: stress produces change in fluid pressure
Fluid-to-solid coupling: change in fluid pressure changes volume of solids

• How are deformation and fluid pressure coupled?
Through mechanical properties and changes in pore pressure
• How to couple deformation and fluid flow?
Couple Darcy’s law and elastic deformation of a porous materials
Boundary conditions on REV matter.
• What can we do with this understanding?
Determine rock (e.g., porosity, compressibility) and transport properties (e.g., permeability)