ABSTRACT
Considering our incomplete knowledge of subsurface systems we have to deal with model uncertainty. In this study, we work with the parameterization or structure of this model to add uncertainty at different levels and, subsequently, update this uncertainty given some observed data using a Bayesian approach. To deal with the high dimensionality of the data space, we use dimension reduction in combination with regression techniques to approximate the posterior probability distribution. We present the application of parameterization by means of a hierarchical Bayesian model and the use of dimension reduction and non-parametric regression to update uncertainty in a subsurface model using geophysical data.

METHODOLOGY
We use a Bayesian approach where we state prior probability distributions of parameters and then obtain their posterior probabilities conditioned on observed data. Uncertain parameters may be considered at different levels using a hierarchical Bayesian model (Wainwright et al., 2014; Feyen & Caers, 2006). The latter can be represented with a graphical model (Fig.2) which explicitly shows conditional dependencies, i.e. the factorization of the joint probability distribution. To obtain samples from this joint probability distribution or its marginals, we sample in topological order (i.e. starting with higher levels).

SYNTHETIC CASE
We apply the method to synthetic GPR cross-borehole data. For illustration purposes we only show the effect of one uncertain discrete parameter: the geological scenario. We considered three different geological scenarios, two of them sampled through multiple-point geostatistics and the third through truncated sequential gaussian simulation (Fig.1a).

Joint probability distribution
\[ p(f|\theta, \eta, \phi, q, m, d) = p(f|\theta) p(\theta|\eta, \phi, q) p(q|m, \phi) p(m|d) \]
1. \( p(f|\theta) \) is sampled through multiple-point geostatistics
2. \( p(\theta|\eta, \phi, q) \) is given by two Gaussian random fields
3. \( p(q|m, \phi) \) is given by considering two different petrophysical relations
4. \( p(m|d, \phi) \) is given by kernel smoothing.

Once a sufficient number of samples is taken, which is dependent on the nature of the problem, we may reduce data dimensionality by means of feature extraction and multi-dimensional scaling. For multi-dimensional scaling we need to define a distance between the samples then an algorithm will project the samples in a lower-dimensional space trying to preserve the original distances (Borg & Groenen, 2005). Finally, in this low-dimensional space we apply kernel smoothing to approximate the posterior probability distribution (Park et al., 2013; Scheidt et al., 2015).

DISCUSSION AND CONCLUSION
We use cross-validation to assess the performance of the method (Fig.5). By taking out one sample (and knowing its true scenario) we compute its posterior probability distribution separately for each geological scenario. Once a sufficient number of samples is taken, which is dependent on the nature of the problem, we can reduce data dimensionality by means of feature extraction and multi-dimensional scaling. For multi-dimensional scaling we need to define a distance between the samples then an algorithm will project the samples in a lower-dimensional space trying to preserve the original distances (Borg & Groenen, 2005). Finally, in this low-dimensional space we apply kernel smoothing to approximate the posterior probability distribution (Park et al., 2013; Scheidt et al., 2015).

REFERENCES