## Advanced inverse modeling and stochastic representations of heterogeneous porous and fractured media

February 12-15, 2138 Géopolis, University of Lausanne

# Instructors: Philippe Renard (University of Neuchâtel) Niklas Linde (University of Lausanne)

The attending students are expected to have a basic working knowledge about Matlab programming. Coffee breaks in the mornings and afternoons will be taken either in the middle of the lectures or between 2-h blocks.

#### **February 12 (Philippe Renard):**

**Lecture 1, 9h00-11h00.** What is geostatistics? Why using it and when? Illustration with a few examples. Overview of the general approach: exploratory data analysis, model identification, forecasts. Reminders of statistics and probability. Quantification of spatial correlation: experimental variogram, covariance and variogram models. Anisotropy. Variable transforms.

**Exercise 1, 11h00-13h00.** Exploration of several data sets: basic statistics. Computation of experimental variograms. Variogram modeling.

**Lecture 2**, **14h00-16h00**. The principles of kriging. The equations in the case of Simple and Ordinary Kriging. Cross validation principle and interpretation. How to infer the variogram, test and select a model.

Exercise 2. 16h00-18h00. Mapping soil contamination using kriging. Testing the model using cross validation. Comparing and testing different models.

#### **February 13 (Philippe Renard):**

**Lecture 3. 9h00-11h00.** Simulations versus estimation. Why do we need simulations? Revisiting the kriging results. Marginal and conditional probabilities, conditional estimation. The multi-Gaussian framework. Normal score transform. Simulating pseudo-random numbers. The Sequential Gaussian Simulation (SGS) algorithm. Overview of different simulation algorithms.

**Exercise 3. 11h00-13h00.** Using SGS to estimate a volume of contaminant and its related uncertainty.

**Lecture 4. 14h00-16h00**. Other simulation techniques, in particular for categorical variables: overview of Sequential Indicator Simulation (SIS), object based techniques, truncated gaussian methods, and introduction to Multiple-Point Statistics (MPS).

**Exercise 4. 16h00-18h00.** Simulation of a categorical variable. Example to model geological heterogeneity: Illustration of the truncated gaussian and MPS approaches.

#### February 14 (Niklas Linde):

Lecture 5, 9h00-11h. Joint and marginal probability density functions. The data and model space. Conditional probabilities and Bayes theorem. Linear vs. non-linear

problems. Prior probability density functions. Likelihood functions. Rejection sampling.

**Lecture 6, 11h-13h.** Markov chain Monte Carlo (MCMC). The Metropolis, Metropolis-Hastings and the extended Metropolis algorithm. Assessing convergence of Markov chain Monte Carlo methods. Acceptance rate and auto-correlation. Reaching the posterior vs. exploring the posterior. Brief introduction to modern Markov chain Monte Carlo methods (parallel tempering, differential evolution, Hamiltonian) and global optimization (simulated annealing; genetic algorithms).

**Exercise 5, 14h00-18h00.** Consider a linear crosshole tomographic GPR example and a multi-Gaussian prior. Implement rejection sampling and MCMC using gradual deformation as model proposal. Assess convergence and compare results with analytical solutions for the mean and the posterior covariance. The code is to be written in Matlab; the forward solver and error-contaminated data will be supplied. If times permit, implement hierarchical Bayes to invert for the standard deviation of the data errors and consider correlated data errors. It will also be possible to work with truncated multi-gaussian fields.

### February 15 (Niklas Linde):

**Lecture 7, 9h00-11h00.** Over-determined linear inverse problems. Least-squares solutions to linear problems. Optimization and uncertainty quantification for linear over-determined problems. Regularization and its effects on model solutions. How to determine the appropriate regularization weight?

**Lecture 8, 11h-13h.** Non-linearity and its implications for parameter estimation. Iterative gradient-based methods. Occam's inversion. Linearized uncertainty and model resolution estimates. Robust and compact inversion. Time-lapse inversion.

Exercise 6, 14h00-18h00. Consider a linear crosshole tomographic GPR example. Implement a deterministic inversion solution together with model appraisal in terms of point-spread functions and posterior covariances. If time permits, implement robust inversion and compact inversion for a test model that is sharp (e.g., an MPS realization).